**The bounds of meta-analytics and an alternative method**

**Supplementary Materials: S1, S2, S3**

**S1. A Derivation of the probability structures of the Higgins statistic** $I^{2}=\frac{(Q - df)}{Q}$

In this section, we first derive the probability density function of and then explore its properties for discussion relevant to the main theme of this article. Let the random number, denote the number of studies on a meta topic available for consideration at a point of time. Realize that the number of studies grows unboundedly in any time depending on the research curiosity of professionals in the topic. Notice that the observable space for the number of studies in meta-analysis is$ y=2,3,...,θ$, where  is an unknown upper-bound parameter. That is, the uniform probability mass function, for the random number of studies to be included in a meta-analysis is so that the total probability is .

By nonnegative variable transformation , we notice that the probability pattern of the transformed variable, is a lower-truncated continuous uniform with the sample space and probability density function (pdf) satisfying the requirements that is non negative and =1.

By applying an additional transformation , we note that not only the observable space changes to  but also the pdf changes to a conjugate satisfying the requirements thatis non-negative and. Figure 1 is the sketch of the pdf .



**Figure 1. The pdf with  in x-axis and  in y-axis.**

Now, we derive the expected value of the transformed random variable, . We note that

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The integral in the above expression is an incomplete gamma function. To find a finite (approximate) result for the integral, we write  due to the Taylor’s series as  because. Notice that the expectation simplifies to , which is sketched in Figure 2.



**Figure 2. in z-axis with in y-axis.**

Now, we proceed to find a finite (approximate) result. By continuation of the same approximation again as seen above, we find that. Substituting the above results in the varianceand simplifying, we obtain thatwhich is sketched in Figure 3.



**Figure 3. z = with in x-axis**

We now proceed to understand the survival function ofand it is  which is sketched in Figure 4 below.



**Figure 4. The survival function  with in x-axis and  in y-axis**

The incremental rate of doing additional studies by the researchers is



which stabilizes at the asymptote when w catches up to . We shall now focus on the conditional probability pattern in the data oriented statistics  for a given , where is another version of . The reparameterizations proceed in steps as follows. The number of studies, y ranges from two to . Note that  ranges from  to one. With  ranges from zero to . We now cast these in terms of our notations. That is, from these transformations, we notice that the degrees of freedom equal to . The probability density function (pdf) of  in terms of a reparametrized version  is.

  (1)

 It is a simple algebraic exercise to prove the requirements,and.

Note that for the sketch of the pdf . ****with  in the y-axis and in x-axis in Figure 5.



**Figure 5. Note that .**

Using the known results  and , the unconditional expected value and variance of the statistic, , are, respectively,

 (2)

and

 (3)