Original Article

Modeling and forecasting of under-five mortality rate in Kermanshah Province of Iran: a time series analysis

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Abstract

Objectives: The target of fourth Millennium Development Goal (MDG-4) is effort to reduce the rate of under-five mortality by two-thirds between 1990 and 2015. Despite substantial progress towards achieving the target of MDG-4 in Iran at the national level, differences at the sub-national levels should be taken into consideration.

Methods: The under-five mortality data available in deputy of public health, Kermanshah University of Medical Sciences, was used in order to perform a time series analysis on monthly under-five mortality rate (U5MR) during 2005 to 2012 in Kermanshah province, the west of Iran. After primary analysis, a seasonal auto-regressive integrated moving average (SARIMA) model was chosen as the best fitting model based on model selection criteria.

Results: The model was assessed and proved to be adequate in describing variations in the data. However, unexpected presence of a stochastic increasing trend and a seasonal component with periodicity of six months in the fitted model are very likely to be a consequence of poor quality of data collection and reporting systems.

Conclusions: The present work is the first attempt in time series modeling of U5MR in Iran, which reveals that improvement of under-five mortality data collection in health facilities and their corresponding systems is a major challenge ahead in fully achieving MGD-4 in Iran. Studies similar to the present work can enhance the understanding of the invisible patterns in U5MR, monitor progress towards MGD-4 and predict the impact of future variations in U5MR.

Keywords: MDG-4; U5MR; Time Series; SARIMA model; Iran

Introduction

The under-five mortality rate (U5MR) is a key pointer of child well-being including health status, and more broadly, an indicator of social and economic progress (1, 2). It is one of the main indicators for assessing and monitoring of progress in child health status with respect to the United Nations' Millennium Development Goal 4 (1-4). The target of fourth Millennium Development Goal (MDG-4) is effort to reduce the rate of under-five mortality by two-thirds between 1990 and 2015 (2, 5). U5MR is defined as the probability (per 1,000 live births) that a child will die before reaching the age of five if subject to current age-specific mortality rates (2). Despite global population growth in recent decades, substantial progress has been made towards achieving MDG-4 in most countries of the world (1, 2, 6), as well as in Iran (6-9). The number of under-five deaths worldwide has declined from nearly 12.7 million in 1990 to 6.3 million in 2013 (2). Consequently, the global U5MR has dropped from almost 90 deaths per 1,000 live births in 1990 to 46 in 2013 (2). However, this progress is not equally distributed at national and sub-national levels (2, 4, 10).

Iran is a country that has experienced considerable reductions in the U5MR over the past decades (11). In Iran, U5MR was estimated as 281 per 1,000 live births in 1970-1975 (6) and declined from 46 per 1,000 live births in 1993 to 25 in 2005 (2). Despite these gains, it has been seen that the decline in U5MR across the country is heterogeneous and unequally distributed among provinces (2, 12, 13). For example, in 2004, the estimate of U5MR using data from Death Registration System of Ministry of Health and Medical Education was 28 per 1,000 live births for the whole country, while it was 32 per 1,000 live births in Kermanshah province, the west of Iran (11). Thus, assessing fluctuations of U5MR at the national level might not be enough and sub-national studies can provide a more detailed understanding of heterogeneity in U5MR across the country. Moreover, since the U5MR is not constant over time, it is also important to understand how the rate evolves over time and explain its stochastic variations using a valid statistical model.

The main purpose of this study is to detect any statistically significant feature in the monthly U5MR in Kermanshah province during 2005 to 2012 and find an appropriate time series model that can adequately explain variations in the monthly U5MR. The study period 2005-2012 was chosen because the child mortality data were available to the authors only for these years at the time of study conduct.

Materials and Methods

Socio-demographic Characteristics

Kermanshah province is one of Iran’s undeveloped provinces that is located in the west of Iran and has 14 counties. Based on the population and housing census data of 2011, the population of the province was 1,945,227 residents (about 2.6% of Iranian population, 78 persons/ km^2 and 69.7% urbanization rate) with mainly Kurdish ethnic background (14).

Ethics Statement

This study is based on the digital file of under-five mortality records available in deputy of public health, Kermanshah University of Medical Sciences. The data were anonymized and de-identified before usage and hence no informed consent was required for this work.

Data source

The monthly frequencies of under-five mortality in Kermanshah province were extracted from the digital file of the province mortality records that occurred during 2005 to 2012 (based on Iranian calendar time) in provincial health jurisdiction. In this mortality digital file, under-five death records were collected from hospitals, health centers and health houses located in the province. There are few studies on sub-national child mortality in Iran, but a published report of Iranian ministry of health shows that in 2010, more than 91% of mortality in neonatal period and more than 63% of mortality in 1-59 months occurred in Iranian hospitals (15). The frequencies were converted to rates per 1,000 live births using the number of under-five mortality in each month as numerator and births in the same month in the province as denominator. Real times of live births were extracted from National Organization for Civil Registration. A sequence of 96 monthly U5MRs were obtained and studied for temporal variations. Through the following steps, a time series analysis has been conducted on the data in order to identify structural patterns in monthly U5MR of Kermanshah province from 2005 to 2012 and a short-term (6 months) prediction has been made. See Appendix 1 and references therein for the technical details on time series analysis.

Data preparation

Box-Cox transformation for assessment of stability in the data variance (with *p*-value=0.20 for the null hypothesis of $\theta = 1$) shows that variance of the U5MR series is constant over time and no variance-stabilizing transformation is required. The Holt–Winters smoothing is applied on the U5MR series to examine any trend in the data. The time series plot of the original and the smoothed U5MR data is shown in Figure 1, upper panel. No seasonal or periodic component is clearly apparent in this plot but the smoothed series suggests an increasing trend, especially in early years of the study period. Prais-Winsten regression also confirms (with t=2.20 and *p*-value=0.030) the presence of a significant linear trend with positive slope. The augmented Dickey-Fuller unit root test accepts (with lags=3, $Z(t)$=-2.51 and approximate *p*-value=0.112) the null hypothesis that the U5MR series has a unit root and which indicates that the increasing trend in data is stochastic and is caused by the effect of random shocks. Therefore, successive differencing could be performed on U5MR to eliminate the unit root and hence the stochastic increasing trend and obtain a zero mean stationary time series. The first difference of U5MR and its smoothed series are shown in Figure 1, lower panel. No trend is recognizable in the smoothed differenced series and the augmented Dickey-Fuller unit root test for the differenced series rejects (with lags=3, $Z(t)$=-6.801 and approximate *p*-value<0.001) the null hypothesis of unit root and confirms that the first difference of U5MR is a stationary time series.

Model Identification and Estimation

The portmanteau Q-test (with Q=91.89 and p-value<0.001) and the Bartlett's periodogram-based test (with B=2.46 and p-value<0.001) for white noise reject the null hypothesis of no serial correlation among observations of the differenced U5MR. To check the structure of such correlations, the sample ACF and PACF plots of the differenced U5MR series are shown in Figure 2, upper panels. According to the plots, only the first lag of the ACF is significant (lay outside the grey 95% confidence bands) and the first few lags of PACF are decaying. The ACF and PACF of a first order moving average, or ARIMA$(0,0,1)$, time series have the same pattern and hence an ARIMA$(0,1,1)$ model is appropriate for the original U5MR series. After fitting the ARIMA$(0,1,1)$ model (with AIC=534.2 and BIC= 541.9) to the U5MR series and obtaining the model residuals, the ACF and PACF of the model residuals are plotted in Figure 2, middle panels. Both ACF and PACF are significant at lag 6 (local spike) indicating that there is a 6-monthly serial correlation in the data that the fitted ARIMA$(0,1,1)$ model is not able to explain it. This could be caused by a periodic (seasonal) component with period $s=6$. Thus, several SARIMA$(p,d,q)\times(P,D,Q)_6$ models with different combinations of (p,d,q) and (P,D,Q) orders are fitted to the U5MR series. Based on corresponding AIC and BIC values of the fitted models, the best fitted model is SARIMA$(0,1,1)\times(0,0,1)_6$ with the smallest value of AIC=526.2and BIC=533.9.

Diagnostic checking

According to Figure 3, there is no major discrepancy between the observed and expected U5MR from the fitted model (upper panel) and the model residuals vary randomly around zero (lower panel). In order to examine the adequacy of the fitted model, the ACF and PACF of the model residuals are shown in Figure 2, lower panels. All lags of ACF and PACF are within 95% confidence bands, indicating there is no correlation structure in the model residuals and hence the fitted SARIMA model is adequate. This is also confirmed by the portmanteau Q-test (with Q=37.4 and p-value=0.59) and Bartlett's periodogram-based test (with B=0.60 and p-value=0.86) for white noise which accepts the null hypothesis of no serial correlation in the model residuals. Moreover, the Shapiro-Wilk test (with W=0.984 and p-value=0.29), and Skewness-kurtosis test (with χ^2=3.64 and p-value=0.16) accept the normality assumption of the model residuals. Finally, the ARCH-LM test for heteroscedasticity in variance of the model residuals accepts (with χ^2=0.004 and p-value=0.951) the null hypothesis of no ARCH effect. Therefore the overall goodness-of-fit of the model is evaluated.

All analyses are performed using Stata SE version 12.0 (StataCorp LP, College Station, TX, USA) (16) and the significance level is chosen at 0.05.

Results

According to the best fitted model, the U5MR at month t, x_t, is determined by

$$x_t = x_{t-1} + z_t - 0.78z_{t-1} - 0.31z_{t-6} + 0.24z_{t-7}$$

where z_t is a white noise process with standard deviation $\hat{\sigma}_z = 3.71$. This means that the U5MR of each month is directly influenced by the U5MR of the first preceding month and independent random shocks of the current, first, sixth and seventh preceding months, with their corresponding coefficients. Detailed description of the estimated model parameters is presented in Table 1. It can be seen that both regular first order moving average parameter and seasonal moving average parameter are statistically significant (p-value<0.05). Therefore, the fitted model suggests three components: a first order difference for removing a stochastic increasing trend, a regular MA(1) term for serial correlation in the data and a seasonal MA(1) for reoccurrence of 6 monthly dependence structures in the data. A significant regular MA(1) component is common in mortality rate time series and indicates that the random fluctuation in the U5MR of each month contributes to the U5MR of the next month. Thus, the presence of this term in the fitted model determines structured uncertainty in the data and provides better prediction. This component of the model is the most persistent and reliable part of the model and we expect that it remain the same over time. The significant increasing trend of the series over the study period contradicts with the national and global decreasing trend. Since the trend is not deterministic, it is likely to be a consequence of rather poor data collection in the early years of the study period. The unexpected presence of a seasonal term with periodicity of six months in the fitted model reveals another issue in the data collection method. Since monthly U5MR data of the province must be reported to the Ministry of Health and Medical Education twice every year (on February and August), it is very likely that unclassified and late reports from different months for some public health facilities be accumulated and mixed up in the final reports. Thus, perhaps like the stochastic trend, the seasonal component of the model, is caused by poor quality of data and may vanish over time if more improvement in the data quality takes place.

The six months ahead (short-term) predictions of U5MR and their corresponding 95% prediction intervals based on the fitted SARIMA model are reported in Table 2. In general, the prediction intervals are wider for time series with stochastic trend (17) and here relatively wide ranges of prediction intervals indicate the high uncertainty associated with the predictions. Using the most recent and more accurate data, such predictions can be used for monitoring and forecasting progress towards MDG-4.

Discussion

In this paper, time series analysis of monthly U5MR data of Kermanshah province in the west of Iran is conducted. According to our findings, the U5MR in Kermanshah province during the study period had a stochastic increasing trend. After primary analysis and applying necessary data adjustments, a $\mathrm{SARIMA}(0,1,1)\times(0,0,1)_6$ was selected as the best fitting model and model assessment and goodness-of-fit tests showed that the model can adequately explain the fluctuations in U5MR. Finally six months ahead of December 2012 predictions and their corresponding prediction intervals were obtained based on the fitted model.

Mortality data sources in developing countries, such as Iran (15, 18, 19), often suffer from various data availability and data quality issues (1, 3, 4) and among these issues under-reporting of deaths and misreporting of ages are common (2, 18). For example, despite considerable efforts for decreasing U5MR trend in the whole country (2, 6-9, 11, 15), the monthly U5MR of Kermanshah province shows a significant increasing trend during the study period. As demonstrated in figure 4, the yearly U5MR of Kermanshah province is clearly lower than the estimated national average in 2005 and then higher than the estimated national average from 2007 to 2012. This discrepancy is most likely caused by under-reporting and misreporting of data in the earlier years and improving quality in data collection and registration coverage of U5MR data in the last years of the study period, which is remarkable for the health care system of Kermanshah province. Although there are some demographic methods for assessing causes and estimation of death under-numeration, but these methods assume balanced population growth (15). Iran has undergone drastic demographic changes during the recent three decades. Population growth had a high rate 30 years ago and afterwards dropped to low values (20, 21). These changes make use of demographic methods for estimation of death under-numeration difficult (15); thus authors were obliged to use the available data for this study.

Besides the possibility that the number of under-five mortality have been under-reported, another limitation of this study is that the available data cover a rather narrow time period of eight years due to restrictions in data availability. The third limitation of the study is that, although the model adequacy was evaluated by various statistical methods, but the capability of the model for forecasting in short time periods and the accuracy of the predicted values by model are not assessed. The uncertainty of the predictions is assessed only by prediction intervals. However, it should be noticed that generating accurate estimates and predictions of child mortality is a considerable challenge for developing countries (1, 2, 4, 10).

Despite its limitations, the most strength of the present study is that it is the first study in Iran that uses time series analysis in order to mathematically model and predict out-of-sample U5MR. However, additional data from other parts of the country is needed to generalize the results to the countrywide levels. In 2015, the fourth Millennium Development Goal target for Iranian under-five mortality rate is 19 deaths per 1,000 live births (2). It is one year before the 2015 deadline of the goal and substantial progress has been made, but the progress remains insufficient to achieve MDG-4, particularly in Kermanshah province. It can be seen from Figure 4 that despite considerable progress in the past two decades to reduce the U5MR in Iran (2, 10, 11), as well as in Kermanshah province (11, 15), the burden of child deaths is higher than the national estimated and expected level in this province.

Conclusions

To achieve MDG-4 on time, reducing in under-five mortality inequities across Iranian provinces is an important priority. It seems that one of the major challenges ahead in fully achieving MGD-4 in Iran is qualitative and quantitative improvement of under-five mortality data collection in health facilities and their corresponding systems. In general, for generating reliable mortality statistics in sub-national levels it is necessary to improve the reporting and registering of child deaths by health facilities and make sure that all child deaths that occur in health facilities are of critical importance. This is also crucial in understanding and measuring the impact and effectiveness of plans and interventions in this field. Since the definition of U5MR indicator has not changed during the recent years, analyzing the invisible patterns and statistical modeling of this indicator within the recent years is valuable and may enable health policy-makers to monitor the progress of Iranian child health status and in order to evaluate the efficacy of health care system. Such findings may be useful as they enhance understanding of the current underling patterns of U5MR and monitoring progress towards MGD-4. Moreover, using studies similar to the present work, it is possible to predict the impact of future changes in this important child health indicator. Surely, with the expansion of time period and improving data collection methods, one can get more accurate results in the future.

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Conflict of interest statement

The authors declare that they have no conflicts of interest for this work.

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Appendix 1: Time series analysis

Time series data

A sequence of observations collected at successive points in time, denoted by $\{x_t\}_{t=1}^{n}$, is called a time series. Typically, the consecutive observations of a time series are serially correlated. A stationary time series has no deterministic or stochastic trend and/or seasonality, its variance is constant over time and it does not contain any unit root (17, 22). Thus, stationary series fluctuate, with constant variance and no periodic cycles, around a constant mean level and do not systematically increase or decrease over time. In general, modeling and forecasting of stationary time series are greatly simpler than nonstationary time series (17, 23). Thus, as a first step, it is necessary to assess the stationarity of the time series and recognize the potential causes of nonstationarity.

Data Preparation

Smoothing techniques are usually used to reduce irregular roughness (random fluctuations) and detect any persistent pattern such as long term trends or cycles in the data (17, 22). The most common types of smoothing techniques are moving averages, single and double exponential smoothing and Holt-Winters exponential smoothing (16, 24). The Holt–Winters method assumes a linear trend with intercept and slope that vary over time and filters out the irregular fluctuations using some smoothing parameters (weights) (22, 24). The presence of a linear trend with non-zero slope in the data can be tested using Prais-Winsten regression method, which uses the generalized least-squares method to estimate the intercept and slope in a linear regression model by allowing errors to be serially correlated (16). The trend can be deterministic or stochastic (17). A time series with a deterministic trend reverts to the trend line in the long run and random shocks (fluctuations) have transitory effects on the level of the series (24). A time series with a stochastic trend or a unit root is not trend reverting and random shocks have permanent effects on the level of the series (22, 23). The hypothesis that the time series contains a unit root (stochastic trend) can be tested using unit root tests like the Augmented Dickey-Fuller test (22, 24). In the case that the presence of a unit root is confirmed, successive differencing can be performed on the data to obtain a zero mean stationary time series. The first lag difference of data, $y_t = \Delta x_t = x_t - x_{t-1}$, can eliminate the unit root (17). In addition, by subtracting the value of an earlier data from the value of a later data, the first lag differencing can also remove a linear deterministic trend in the data (22, 24). For more complex situations, higher orders of lag differencing might be needed to eliminate several unit roots and/or non-linear deterministic trends (17). Besides trend, nonconstant variance can also cause nonstationarity (17). Box-Cox transformation with parameter θ is defined as $\left(x_t^{\theta} - 1\right)/\theta$ and can be applied on time series in which variance changes over time to stabilize the variance (17).

Model Identification and Estimation

The Autoregressive Integrated Moving Average (ARIMA) model by Box and Jenkins is the most well-known model for a wide variety of time series (24). An ARIMA(p, d, q) model does not require the original time series x_t to be stationary but it assumes that after d times differencing, the new series $y_t = \Delta^d x_t$ becomes a stationary zero mean time series. It also assumes that the current value of the differenced series y_t is a linear combination of p previous values $y_{t-1}, y_{t-2}, \dots, y_{t-p}$ and $q+1$ values $z_t, z_{t-1}, \dots, z_{t-q}$ of a white noise process (independent and identically distributed random shocks with zero mean and common variance σ_z^2); that is

$$y_t = \varphi_1 y_{t-1} + \varphi_2 y_{t-2} + \cdots + \varphi_p y_{t-p} + z_t + \theta_1 z_{t-1} + \cdots + \theta_q z_{t-q}.$$

The autoregressive order p is the number of terms in the model that describe the direct dependency among successive observations and the moving average order q is the number of persistent random shocks that describe indirect dependency among successive observations (24). They are identified by examining patterns in plots of the autocorrelation function (ACF) and partial autocorrelation function (PACF) of the differenced series y_t (22, 24). After identifying p and q, the model parameters $\varphi_1, \dots, \varphi_p$, $\theta_1, \dots, \theta_q$ and σ_z^2 are estimated using the maximum likelihood estimation method (17). If several values for p and q are seems plausible, model selection criteria such as Akaike Information Criterion (AIC) and the Schwarz's Bayesian Information Criterion (BIC) can be used to select the best fitting model (23).

Seasonal or periodic cycles produce local spikes at specific lags in plots of ACF and PACF (22, 24). Any ARIMA model can be extended to include seasonality which is called a Seasonal Autoregressive Integrated Moving Average (SARIMA) model. A SARIMA$(p, d, q) \times (P, D, Q)_s$ model consists of two multiplicative components: a regular ARIMA(p, d, q) component and a seasonal component with autoregressive order P, moving average order Q and difference order D (17, 25, 26). The seasonal component is responsible for recurrence of a recognizable pattern after the seasonal period s. The general model equation of a SARIMA$(p, d, q) \times (P, D, Q)_s$ model can be found in (17) which has p regular autoregressive parameters, q regular moving average parameters, P seasonal autoregressive parameters and Q seasonal moving average parameters. Similar to the regular ARIMA model, ACF and PACF plots and AIC and BIC criteria can be used to identify the seasonal orders (P, D, Q), though in this case the situation is more complicated (17). Using the maximum likelihood method, all $p + q + P + Q + 1$ regular and seasonal parameters of the model can be estimated (22, 23).

Diagnostic Checking

After fitting the desired SARIMA model to the observed time series, the model residuals are obtained by subtracting the expected values of observations under the model, $\hat{x}_t$, from the true observed values, x_t; that is $r_t = x_t - \hat{x}_t$ (24). The model residuals contain any features in the data that are not explained by the fitted model. Thus, if the fitted model adequately explains all statistically significant features of the data, then the model residuals must behave as a white noise process (22). In other word, when the fitted model is good enough for the data, then the model residuals are expected to have no significant feature, including trend, seasonality, variance heteroscedasticity and serial correlation. Furthermore, the maximum likelihood estimation method is based on the assumption of normally distributed random shocks z_t which implies that the model residuals be normally distributed (17). Plots of ACF and PACF of the model residuals are useful in assessing no significant serial correlation in the model residuals. If the fitted model is adequate, both ACF and PACF of residuals should have no structure to identify (23, 24). In addition, portmanteau Q-test (goodness-of-fit statistic) and Bartlett's periodogram-based test for white noise can be also used to test the existence of serial correlation in the model residuals (22). The adequacy of the normality assumption of the model residuals can be checked using normality tests such as Shapiro-wilk, and Skewness-kurtosis tests (24, 27). Finally, the heteroscedasticity (deviation from constant variance assumption) in the model residuals can be examined by performing ARCH-LM test for autoregressive conditional heteroscedasticity (ARCH) effects (22, 23).

Prediction and Prediction Intervals

Once a suitable model has been fitted to the data, the known structure of the model can be used to predict future values of the time series based on previously observed values with a recursive method (22). Prediction error, the difference between the actual and the predicted values, is inevitable and hence the uncertainty of the predictions must be considered; the farther the prediction beyond the observed values, the less reliable the prediction. For this reason, an interval that with a certain probability (for example 95%) will contain the actual future value, called prediction interval, is also reported for each prediction (24). The prediction interval (PI) is always wider than the corresponding confidence interval because of the added uncertainty involved in predicting a random variable versus its mean (22, 23).

Tables

Table 1: Detailed description of estimated parameters of the fitted SARIMA(0, 1, 1)×(0, 0, 1)$_6$ model to Kermanshah province monthly U5MR data.

Log likelihood = -260.11			Wald χ^2=	139.91	*p*-value < 0.0001	
Model parameters	Estimate	Standard Error	Z-statistic	*p*-value	95% Confidence Interval	
regular MA(1)	-.779	0.0719	-10.83	<0.001	-0.920	-0.638
seasonal MA(1)	-0.306	0.1079	-2.84	0.005	-0.518	-0.095
σ_z	3.707	0.2919	12.70	<0.001	3.135	4.279

Table 2: The 6 month ahead predicted values of U5MR with corresponding 95% lower and upper prediction intervals based on the fitted SARIMA(0, 1, 1) $\times$(0, 0, 1)$_6$ model.

Time	Predicted value	95% Prediction intervals	
		Lower prediction limit	Upper prediction limit
2013m1	24.68	17.40	31.96
2013m2	22.05	14.60	29.49
2013m3	23.69	16.08	31.29
2013m4	23.35	15.59	31.12
2013m5	23.89	15.97	31.81
2013m6	23.43	15.35	31.50

Figures

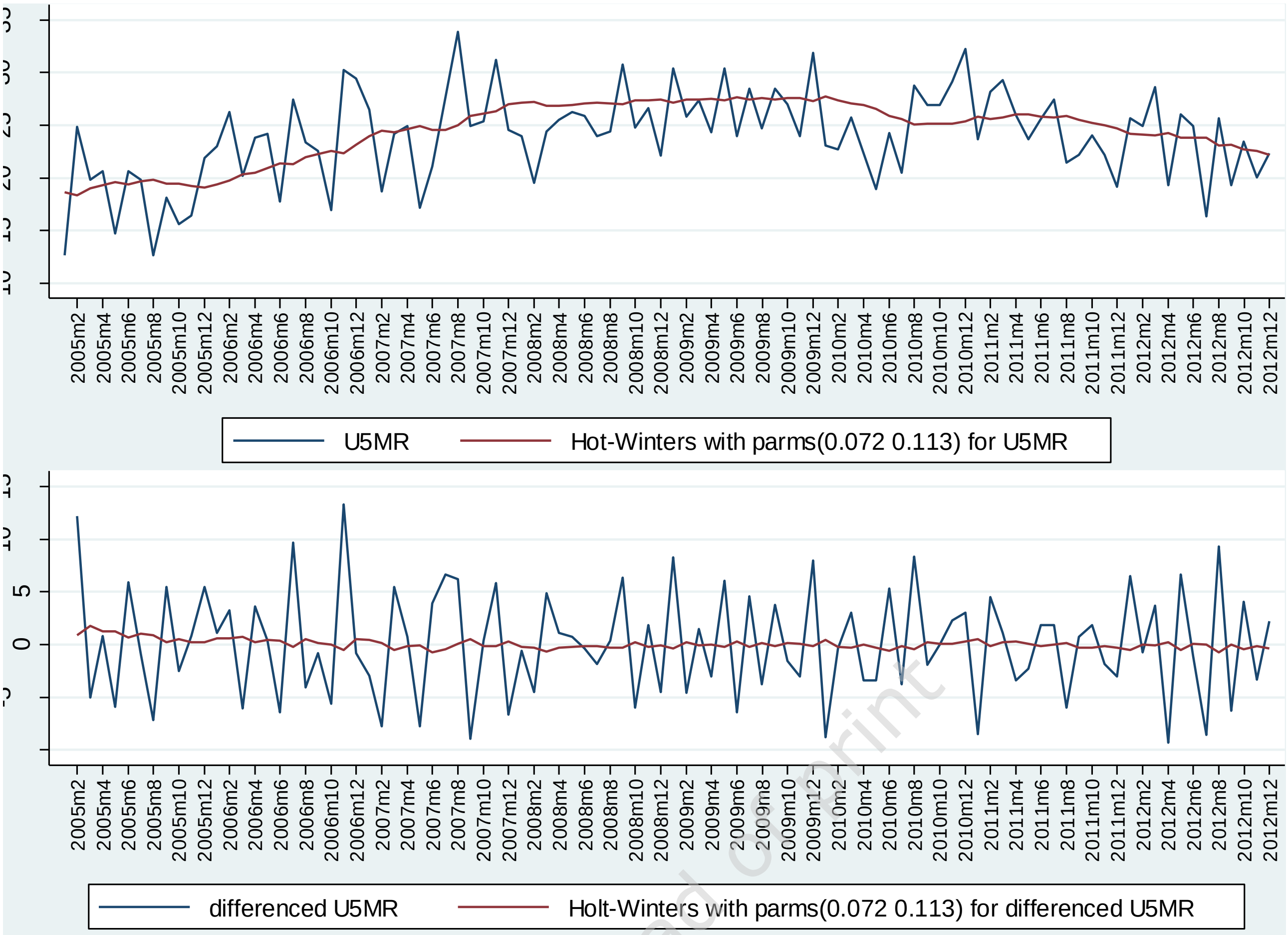

Figure 1: Time series plot of the original and smoothed U5MR data (upper panel) and differenced U5MR (lower panel).

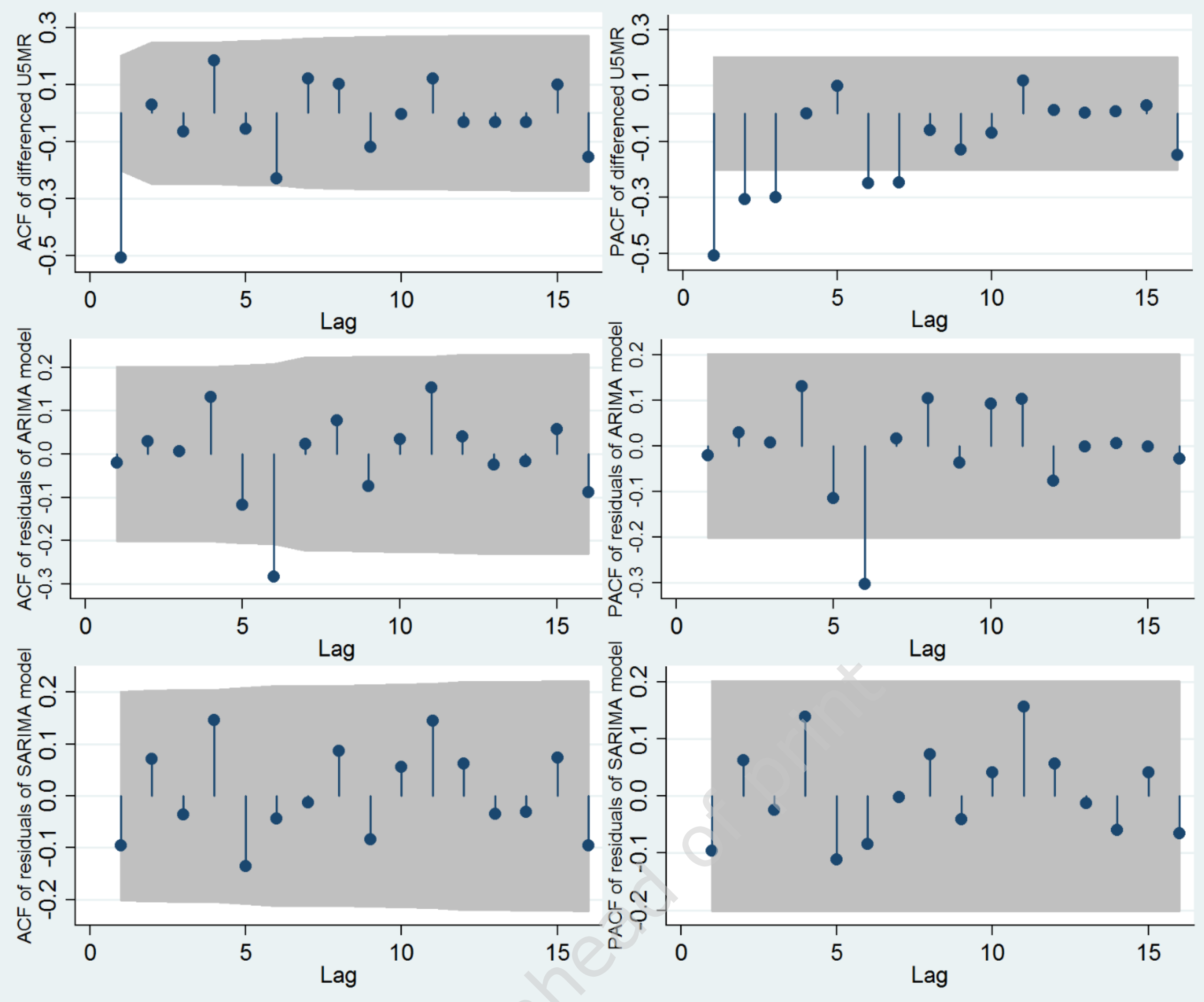

Figure 2: ACF (left) and PACF (right) plots of the differenced U5MR series (upper panels), the residual of the fitted ARIMA(0, 1, 1) model (middle panels), and the residuals of the fitted SARIMA$(0,1,1)\times(0,0,1)_6$ model (lower panels).

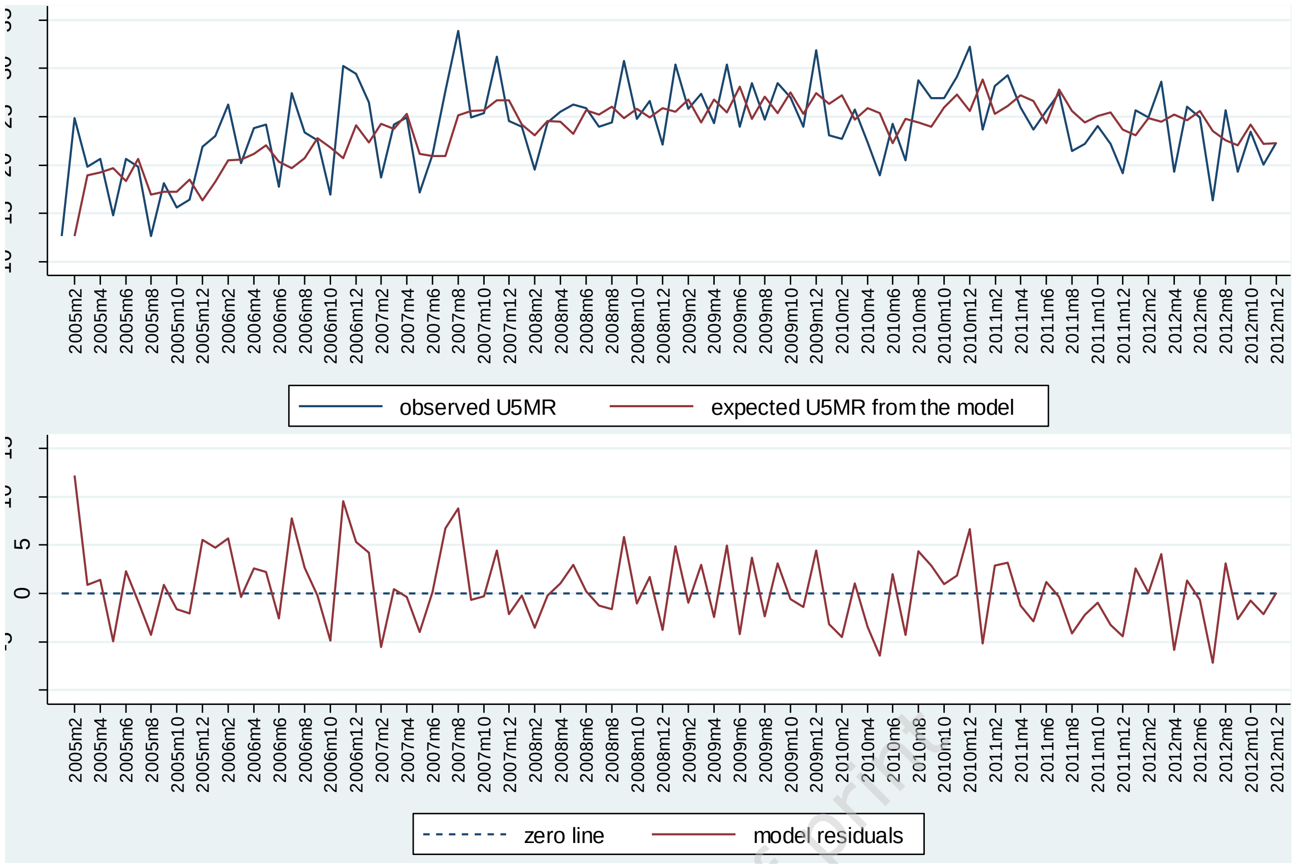

Figure 3: The observed and expected U5MR from the fitted model (upper panel) and model residuals (lower panel).

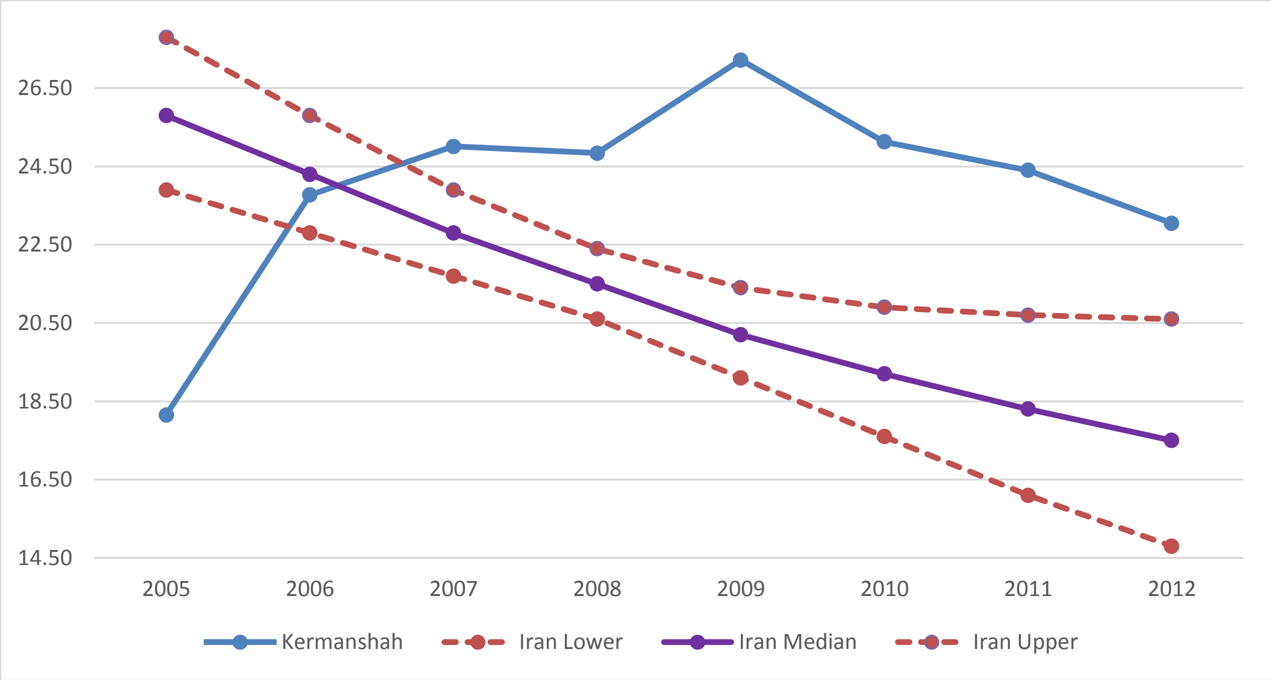

Figure 4: Yearly under-five mortality rate (deaths per 1,000 live births) for Kermanshah province from 2005 to 2012 (resulted from this study) and lower, middle and upper estimates of yearly under-five mortality rate for Iran (extracted from child mortality database, CME Info (www.childmortality.org)).